Notes and Examples for Part 3:

- Properties of Logarithims

- Solving Logarithmic Equations

PreCalculus
Notes

Name: ______________
Date: ______________

Logarithmic Functions – Day 1

Vocabulary:

1. A <u>logarithmic</u> function has a variable as a base and a number as the power. It is written as <u>$\log_a n = x$</u>, where a is the <u>base</u>, x is the <u>exponent</u> and n is the <u>solution</u>.

   $\log_a n = x \qquad a^x = n$

2. A <u>logarithmic</u> function has a base that is a real number and the exponent is a variable.

3. A logarithm is another way of writing an <u>exponent</u>. $Log_b n = p$ Is the same as <u>$b^p = n$</u>, where b is the <u>base</u>, n is the <u>solution</u> and p is the <u>exponent/power</u>.

4. Logarithms can also be written as a <u>function</u>: <u>$y = \log_b n$</u> where $b > 0$ and $b \neq 1$.

   a. The logarithmic function $y = \log_a x$ where $a > 0$ and $a \neq 1$ is the <u>inverse</u> of the exponential function $y = a^x$. So, $y = \log_a x$ if and only if <u>$x = a^y$</u>.

Examples:

Write each equation in exponential form.

1. $\log_{27} 3 = \frac{1}{3}$

   $27^{\frac{1}{3}} = 3$

2. $\log_{16} 4 = \frac{1}{2}$

   $16^{\frac{1}{2}} = 4$

Write each equation in logarithmic form.

3. $4^2 = 16$

   $\log_4 16 = 2$

4. $2^{-3} = \frac{1}{8}$

   $\log_2 \frac{1}{8} = -3$

Evaluate each expression.

5. $\log_4 64 = x$

$4^x = 64$

$4^x = 4^3$

$\boxed{x = 3}$

64
4 16
4 4

6. $\log_7 \frac{1}{343} = x$

$7^x = \frac{1}{343}$

$7^x = \frac{1}{7^3}$

$7^x = 7^{-3}$

$\boxed{x = -3}$

343
7 49
7 7

7. $\log_2 .5 = x$

$2^x = \frac{1}{2}$

$2^x = 2^{-1}$

$\boxed{x = -1}$

8. $\log_8 32 = x$

can't get 32 in terms of 8, so put both in terms of 2.

32
8 4
2 4 2 2
2 2

8
4 2
2 2

$8^x = 32$

$2^{3x} = 2^5$

$3x = 5$

$\boxed{x = \frac{5}{3}}$

9. $\log_4 \frac{1}{256} = x$

$4^x = \frac{1}{256}$

$4^x = \frac{1}{4^4}$

$4^x = 4^{-4}$

$\boxed{x = -4}$

256
16 16
4 4 4 4

10. $\log_{16} \frac{1}{8} = x$

$16^x = \frac{1}{8}$

$2^{4x} = \frac{1}{2^3}$

$2^{4x} = 2^{-3}$

$4x = -3$

$\boxed{x = \frac{-3}{4}}$

16
4 4
2 2 2 2

8
4 2
2 2

Required Practice – #'s 1 – 13 on the Logarithms Worksheet

Logarithmic Functions – Day 2

Properties of Logarithms

Equality Property of Logarithms:	If $\log_b m = \log_b n$, then $m = n$.
Product Property of Logarithms:	For all positive numbers m, n, and b where $b \neq 0$, $\log_b (mn) = \log_b m + \log_b n$.
Quotient Property of Logarithms:	For all positive numbers m, n, and b where $b \neq 0$, $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$.
Power Property of Logarithms:	For all positive numbers m, n, and b where $b \neq 0$, $\log_b m^p = p \log_b m$.

Examples:

Solve each equation.

1. $\log_b \frac{1}{25} = -2$

$b^{-2} = \frac{1}{25}$

$b^{-2} = 5^{-2}$

$\boxed{b = 5}$

2. $\log_b 15625^{\frac{1}{6}} = \frac{1}{3}$

$b^{\frac{1}{3}} = 15625^{\frac{1}{6}}$

$\left(b^{\frac{1}{3}}\right)^3 = (5)^3$

$\boxed{b = 125}$

3. $\log_{10}(2x+5) = \log_{10}(5x-4)$

$2x + 5 = 5x - 4$

$9 = 3x$

$\boxed{3 = x}$

4. $\log_7 n = \frac{2}{3} \log_7 8$

$\log_7 n = \log_7 8^{2/3}$

$n = 8^{2/3}$

$\boxed{n = 4}$

5. $2\log_6 4 - \frac{1}{4}\log_6 16 = \log_6 x$

$$\log_6 4^2 - \log_6 16^{1/4} = \log_6 x$$

$$\log_6 \frac{4^2}{16^{1/4}} = \log_6 x$$

$$\frac{4^2}{16^{1/4}} = x$$

$$\frac{16}{2} = x \qquad \boxed{x = 8}$$

~ plug in and make sure you don't get a negative!

6. $\log_3(4x+5) - \log_3(3-2x) = 2$

$$\log_3 \frac{(4x+5)}{(3-2x)} = 2$$

$$3^2 = \frac{4x+5}{3-2x}$$

$$9(3-2x) = 4x+5$$

$$27 - 18x = 4x + 5$$

$$+18x \quad +18x$$

$$27 = 22x + 5$$

$$-5 \qquad -5$$

$$22 = 22x$$

$$\boxed{1 = x}$$

7. $\log_6(a^2+2) + \log_6 2 = 2$

$$\log_6 2(a^2+2) = 2$$

$$6^2 = 2(a^2+2)$$

$$36 = 2a^2 + 4$$

$$-4 \qquad -4$$

$$\frac{32}{2} = \frac{2a^2}{2}$$

$$16 = a^2$$

$$\boxed{a = \pm 4}$$

~ check

Required Practice – #'s 14 – 30 on the Logarithms Worksheet

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Notes

Name: ______________
Date: ______________

Using Common Logarithms to Solve Equations

Review:

Rewrite 7,000,000 in scientific notation. 7×10^6

Vocabulary:

1. When working with a logarithm and no base is indicated, the base is 10.
2. When a logarithm has a base of 10, it is called a common logarithm, and is made up of two parts…
   a. The characteristic is the exponent of 10 that is used to write the number in scientific notation.
   b. The mantissa is the logarithm of a number between 1 and 10.
   c. So, in log (7,000,000), or log(7×10^6), the mantissa is 7 and the characteristic is 6.
3. A logarithm does not have to have a base of 10. When it does not have a base of 10, the change of base formula can be used to find the value. The change of base formula states that if a, b, and n are positive numbers and neither a nor b is 1, then

   $\log_a n = \dfrac{\log_b n}{\log_b a}$. ~> we can use this to change to base 10.
4. In the equation $\log_b n = p$, if p is known but n is unknown, then n is called the antilog of p, written antilog(p). So, $n = anti\log(p)$.

   **But remember, the inverse of a logarithmic function is still an exponential function!
5. The properties of logarithmic functions apply to common logarithms.

Examples:

Use your calculator to evaluate each expression. Round each answer to the nearest ten-thousandth.

$\log(x) = a$

1. $\log 424 = 2.6274$

2. antilog 4.8740

$\log_{10} x = 4.8740$

$10^{4.8740} = x$ $\boxed{x = 74816.95}$

3. $y = \log(3 \cdot 5^3) = 2.5740$

4. $\log \frac{13^3}{7} = 2.4967$

watch your entry!

Find the value of each logarithm using the change of base formula. Round each answer to the nearest ten-thousandth. Remember, the change of base formula is:

5. $\log_{12} 18$

$\frac{\log_{10} 18}{\log_{10} 12} = \frac{1.2553}{1.0792}$

$\boxed{\approx 1.1632}$

6. $\log_8 172$

$\frac{\log_{10} 172}{\log_{10} 8} = \frac{2.2355}{0.9031}$

$\boxed{\approx 2.4754}$

Solve each equation or inequality. Round each answer to the nearest ten-thousandth.

7. $5^{4x} = 73$

$\log 5^{4x} = \log 73$

$4x \log 5 = \log 73$

$4x = \frac{\log 73}{\log 5}$

$\frac{4x}{4} = \frac{2.6658}{4}$

$\boxed{x = 0.6665}$

8. $2^{x-1} = 5^{x-2}$

$$\log 2^{x-1} = \log 5^{x-2}$$
$$(x-1)\log 2 = (x-2)\log 5$$
$$\frac{x-1}{x-2} = \frac{\log 5}{\log 2}$$
$$\frac{x-1}{x-2} = 2.3219$$

$$x-1 = (x-2)(2.3219)$$
$$x-1 = 2.3219x - 4.6436$$
$$+1 \qquad\qquad +1$$
$$x = 2.3219x - 3.6436$$
$$-2.3219x \quad -2.3219x$$
$$-1.3219x = -3.6436$$
$$\boxed{x = 2.75633}$$

* graph to see the intersection

9. $4.3^x < 76.2$

$$\log 4.3^x < \log 76.2$$
$$x \log 4.3 < \log 76.2$$
$$x < \frac{\log 76.2}{\log 4.3}$$
$$\boxed{x < 2.9709}$$

10. $8^{y+4} > 15$

$$\log 8^{y+4} > \log 15$$
$$(y+4)\log 8 > \log 15$$
$$y+4 > \frac{\log 15}{\log 8}$$
$$y+4 > 1.3023$$
$$-4 \qquad -4$$
$$\boxed{y > -2.6977}$$

Practice Problems on Page 731
Mandatory: 34 – 44 even, Suggested: 35 – 43 odd
and 55-57

PreCalculus
Notes

Name: ________________
Date: ________________

Natural Logarithms

Vocabulary

1. Logarithms that have a base of e are called natural logarithms and are abbreviated as $\ln(x)$. In other words, $\log_e(x) = \ln(x)$.

   a. e is a constant with a value of 2.71828....

2. Logarithms with a base other than e can be converted to natural logarithms using the change of base formula: $\log_a(n) = \frac{\log_e n}{\log_e a} = \frac{\ln(n)}{\ln(a)}$.

3. Sometimes you know the natural logarithm of a number x and you must find x.

   If ln(x) = a, then x is the anti ln of a.

   ($\text{anti}\ln(x) = e^x$)

4. The properties of logarithms can be used to solve logarithmic equations and inequalities with natural logarithms also.

What is $\ln e$? $\ln e = 1$ $e^1 = e$

Examples

Evaluate each expression. Round to the nearest ten-thousandth.

1. ln .0089

   -4.7217

2. $\ln\left(\frac{1}{0.32}\right)$

   1.1394

3. *anti* ln(-.7831)

   0.4570

Use natural logarithms to solve each equation. Round to the nearest ten-thousandth.
**You can plug your x back in to check you answer!

6. $5^{2x} = 7^{x+1}$

$\ln 5^{2x} = \ln 7^{x+1}$

$\frac{2x \ln 5}{\ln 5} = \frac{(x+1)\ln 7}{\ln 5}$

$2x = (x+1)(1.2091)$

$2x = 1.2091x + 1.2091$

$-1.2091 \quad -1.2091x$

$\frac{0.7909x}{0.7909} = \frac{1.2091}{0.7909}$

$\boxed{x = 1.5288}$

7. $5^{x+3} > 10^{x-6}$

$(x+3)\ln 5 > (x-6)\ln 10$

$x+3 > (x-6)\frac{\ln 10}{\ln 5}$

$x+3 > (x-6)(1.4307)$

$x+3 > 1.4307x - 8.5842$

$-x + 8.5842 \quad -x \quad +8.5842$

$\frac{11.5842}{0.4307} > \frac{0.4307x}{0.4307}$

$\boxed{26.8962 > x}$

8. $\frac{7.2}{-28.8} = \frac{-28.8 \ln(x)}{-28.8}$

$-0.25 = \ln_e x$

use the exponential!

$e^{-0.25} = x$

$\boxed{0.7788 = x}$

9. $25 > e^{0.2t}$

$\ln 25 > 0.2t \ln e$

$\frac{\ln 25}{0.2} > \frac{0.2t}{0.2}$

$\boxed{16.094 > t}$

Required Practice: Worksheet Ln problems
(#2, 3, 7, 9, 12, 13, 15)